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NEW NUMERICAL METHOD NK1 FOR CALCULATING SINGLE INTEGRALS

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ABSTRACT

The main objective of the research is to find a new numeric formula to calculate the values of single definite integrals with continuous integrands or these which its integrands are continuous but which is improper at the derivative or whose integrands are improper at one or both boundary of integration by depending on the Mid-point rule which is one of the Newton- Cotes integrative rules, and find the corresponding correction terms when the function of integral is continuous, discontinuous or its derivative is not continuous in the integral region. It's also aim to compare the results of this rule with the results of the Newton-Cotes rules (Trapezoidal, Mid-point and Simpson), and we will show the extent of preference over the above methods, also to explain the importance of the correction limits associated with the new base by using with it Romberg acceleration, where we obtained good results clearly in improving the values of integrations in terms of accuracy and the number of partial periods.

Key word: *Numerical integration, Newton-Cotes, Correction terms, Romberg acceleration.*

INTRODUCTION

The numerical integration study how to find the approximate value of any definite integral. The application of numerical integration its beginnings dates back to find the area of a circle in the Greek quadrature mothed by dividing the circle into regular polygons, by this method, Archimedes was able to find the upper and lower limits of the mathematical constant value. Numerical analysis has been characterized by the creation of a various methods to find an approximate solutions to certain mathematical issues by an efficient manner, the efficiency of which depends on the accuracy and the ease with which they can be implemented. The modern numerical analysis is the wide field of applied analysis. Since the sixteenth century, many analytical methods have begun to emerge, from these methods are the use of the fundamental theorem of integration calculus, infinite series, functional relationships and integrative transformations.

Many researchers have worked in field of the single definite integrals because of their importance in calculating the areas, volumes of rotational objects, length of a plane curve, moments, center of mass, and liquid pressure, Ayres Frank JR [2].A book (Methods of Numerical Integration) it's a good reference which was written by Davis and Phillip Rabinowitz in 1975 [8], as well as the works of Fox [6] in 1967, Fox and Hayes [5] in 1970 and Shanks [10] in 1972 who dealt with this subject from several aspects.

In 2011, Mohamed et al. [7] presented a suggested a numerical method to calculating single integrals taking advantage from Trapezoidal and Mid-point rules.

In 2016, Alsharify et al. [1] introduced a new numerical method dependent on Trapezoidal and Simpson's rules.

NEWTON-COTES FORMULAS

In general, the Newton-Cotes rules can be written as follows

$$J = \int_{t_0}^{t_{2p}} \delta(r)dr = \Omega(t) + C_\Omega(t) + R_\Omega \qquad \cdots (1)$$

by assuming that the number of partial periods is $2p$, where $\Omega(t)$ is the expression of Lagrangion-Approximation for the value of the integral J by using the numerical rule Ω, $C_\Omega(t)$ is the correction terms for $\Omega(t)$, and R_Ω is the remainder which is related after using a serval terms of $C_\Omega(t)$ and $t = (t_{2p} - t_0)/2p$, Syfi [11].

An even number of partial periods must be selected because the Simpson's rule applies only if the number of periods of fragmentation is even and this does not effect of the other rules.

The general formulas of $\Omega(t)$ is :

$$\Omega(t) = t(\lambda\delta_0 + \bar{\lambda}\delta_1 + \bar{\bar{\lambda}}\delta_2 + \cdots + \bar{\lambda}\delta_{2p-1} + \lambda\delta_{2p}) \qquad \cdots (2)$$

where $\delta_i = \delta(r_i), r_i = r_0 + it, i = 0,1,2,\cdots,2p$ and the wattage transactions take order $\lambda, \bar{\lambda}, \bar{\bar{\lambda}}, \cdots, \bar{\lambda}, \lambda$.

So $2p + 1$ was needed as a pivotal points, and to simplify the equation (2) take

$$\bar{\lambda} = 2(1 - \lambda) \text{ and } \bar{\bar{\lambda}} = 2\lambda \qquad \cdots (3)$$

Now, if $\lambda = 1/2$ get Trapezoidal rule which is:

$$T(t) = \frac{t}{2}\left[\delta_0 + \delta_{2p} + 2\sum_{i=1}^{2p-1} \delta_i\right] \qquad \cdots (4)$$

When $\lambda = 1/3$ get Simpson's rule which is:

$$S(t) = \frac{t}{3}\left[\delta_0 + \delta_{2p} + 4\sum_{i=1}^{2p-1} \delta_{2i-1} + 2\sum_{i=1}^{2p} \delta_{2i}\right] \qquad \cdots (5)$$

When $\lambda = 0$ get Mid-point rule which is:

$$M(t) = t\sum_{i=1}^{2p-1} \delta_{i+\frac{t}{2}} \qquad \cdots (6)$$

THE NEW RULE

Theorem: The approximation value of the integral $J = \int_{r_0}^{r_m} \delta(r)dr$ is

$$J = \frac{t}{3}\left[4\sum_{i=1}^{m}\delta\left(r_0 + \frac{2i-1}{2}t\right) - 2\sum_{i=1}^{\frac{m}{2}}\delta(r_{2i-1})\right] \text{ where } m = (r_m - r_0)/t\,.$$

Proof: Since the number of partial periods (h) is even, then put $h = 2t$, so :

$$J_t = t\left[\delta\left(r_0 + \frac{t}{2}\right) + \delta\left(r_0 + \frac{3t}{2}\right) + \delta\left(r_0 + \frac{5t}{2}\right) + \cdots + \delta\left(r_0 + \frac{(2m-1)\,t}{2}\right)\right] \quad \cdots(7)$$

$$J_h = 2t[\delta(r_1) + \delta(r_3) + \delta(r_5) + \cdots + \delta(r_{m-1})] \quad \cdots(8)$$

But $J = J_t + \dfrac{J_t - J_h}{\left(\frac{h}{t}\right)^2 - 1}$, Dorn[3] $\cdots(9)$

From the equations (7), (8) and (9) deduce that:

$$\begin{aligned} J &= t\left[\delta\left(r_0 + \frac{t}{2}\right) + \delta\left(r_0 + \frac{3t}{2}\right) + \delta\left(r_0 + \frac{5t}{2}\right) + \cdots + \delta\left(r_0 + \frac{(2m-1)\,t}{2}\right)\right] \\ &+ \frac{t}{3}\left[\delta\left(r_0 + \frac{t}{2}\right) + \delta\left(r_0 + \frac{3t}{2}\right) + \delta\left(r_0 + \frac{5t}{2}\right) + \cdots + \delta\left(r_0 + \frac{(2m-1)\,t}{2}\right)\right] \\ &- \frac{2t}{3}[\delta(r_1) + \delta(r_3) + \delta(r_5) + \cdots + \delta(r_{m-1})] \end{aligned}$$

Therefore, $$J = \frac{t}{3}\left[4\sum_{i=1}^{m}\delta\left(r_0 + \frac{2i-1}{2}t\right) - 2\sum_{i=1}^{\frac{m}{2}}\delta(r_{2i-1})\right] \quad \cdots(10)$$

The formula (10) will call it the base of Nada Al-Karamy-1 and sign it with the symbol NK1.

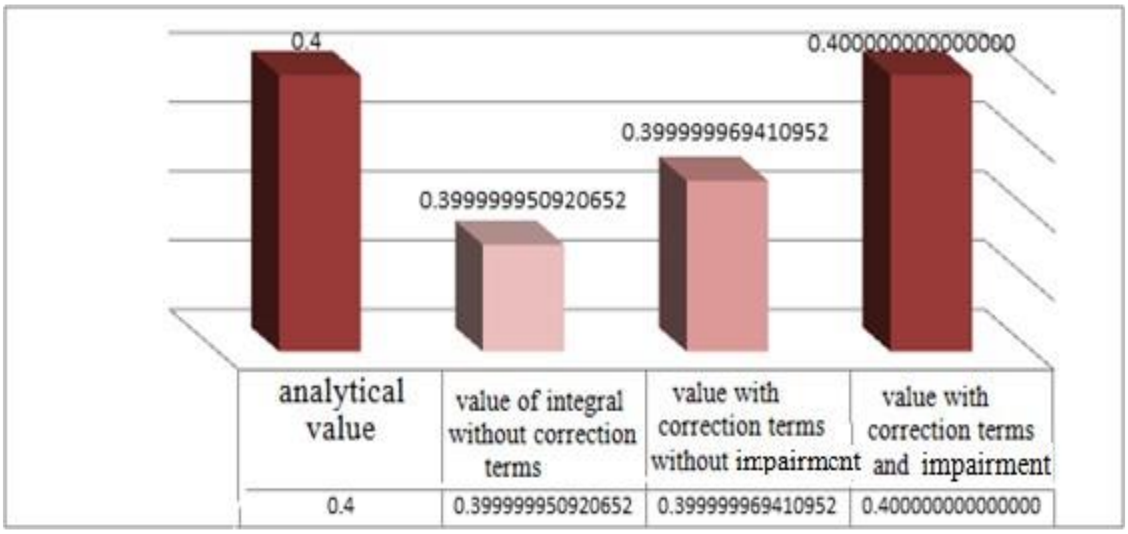
To confirm the validity of the derived rule, eight examples were taken which is different in behavior of function and found the integrals values by using this method and compare these values with the values by using Newton-Cotes rules, table (1) shows that the approximate values obtained using the new rule are closer to the analytical values of the values when using the Newton-Cotes formulas.

No	Integral and it's analytical value	$NK1/m$	S/m	M/m	T/m	Type of integration
1	$\int_1^2 \frac{\ln(x)}{x}dx$ 0.24022650695910	0.240226507 / 100	0.240226507 / 108	0.240227 / 198	0.2402265 / 1164	Continuous integrand over the interval of integration
2	$\int_0^1 x\sin^2(x)dx$ 0.199693997861972	0.199693998 / 100	0.199693998 / 140	0.199694 / 368	0.199694 / 520	
3	$\int_1^2 x^2\sin^2(x)dx$ 2.165316129709335	2.165316129 / 96	2.165316129 / 108	2.165316 / 480	2.1653161 / 1466	
4	$\int_1^2 xe^x + \ln(x)dx$ 7.775350460050541	7.775350460 / 122	7.775350460 / 132	7.775350 / 840	7.77535 / 546	
5	$\int_0^1 \sqrt{x^2+x}\,dx$ 0.840316775024936	0.840317 / 1026	0.840317 / 4434	0.840317 / 1898	0.840317 / 8274	The function is infinite at one of the limits integration
6	$\int_1^2 \sqrt{x-1}\ln(x)dx$ 3.03789458065588	3.03789458 / 764	3.03789458 / 1012	3.037895 / 2094	3.037895 / 872	
7	$\int_1^2 \frac{\ln(x)}{\sqrt{x-1}}dx$ 0.527887014709684	0.527887 / 1340	0.527887 / 2920	0.527887 / 2504	0527887 / 5462	
8	$\int_0^1 e^{\sqrt{2-2x}}dx$ 2.703764092328159	2.7038 / 54	2.7038 / 406	2.7038 / 94	2.7038 / 748	

Table (1) Compare between Nada Al-Karamy-1 rule and Newton-Cotes formulas

THE CORRECTION LIMITS OF NK1

The error formulas in numerical solutions are of great importance in improving the results, to show that take the integral $\int_0^1 \sqrt{x^3}\,dx$ which analytical value is 0.4, but by using NK1 with $m = 128$ the approximated value is correct for seven decimal places, while we get best value with correction terms by ignoring the improper at derivatives on the other hand the value was obtained equal to the analytical value when the correction limits based on the improper at the derivative was applied, as shown in the chart below.



To find correction terms $C_\Omega(t)$ of NK1 rule, define the Operators Δ, ∇, E and D as follow

$\Delta\delta(k) = \delta(t+k) - \delta(k)$

$\nabla\delta(k) = \delta(k) - \delta(k-t)$

$E\delta(k) = \delta(t+k)$

$D\delta(k) = \delta'(k)$

So $C_\Omega(t) = D^{-1}(E^{2p} - 1)\delta_0 - \Omega(t)$ $\cdots(11)$

By assume that m is an even number such that $m = 2p$, from the equations (2) and (3) get:

$$t^{-1}\Omega(t) = \left[\left(\lambda(1+E^{2p}) + \bar{\lambda}(1+E^2+\cdots+E^{2p-2}) + \bar{\bar{\lambda}}(1+E^2+\cdots+E^{2p-4})\right)\right]\delta_0$$

$$t^{-1}\Omega(t) = \left[\frac{\lambda(E-1)^2 + 2E}{E^2-1}\right](E^{2p}-1)\delta_0 \qquad \cdots(12)$$

From the equations (11) and (12) conclude that:

$$C_\Omega(t) = t\left[\frac{(tD)^{-1} + \lambda(E-1)^2 + 2E}{E^2-1}\right](\delta_{2p} - \delta_0) \qquad \cdots(13)$$

Since $E = \Delta + 1$, then in term of forward differences Δ

$$C_\Omega(t) = t\left[\lambda\left(\frac{1}{2}\Delta - \frac{1}{4}\Delta^2 + \frac{1}{8}\Delta^3 - \cdots\right) + \left(-\frac{1}{6}\Delta + \frac{1}{12}\Delta^2 - \frac{13}{360}\Delta^3 + \frac{1}{80}\Delta^4\cdots\right)\right]\delta_0 \cdots(14)$$

By the same way equation (13) can be written in term of backward differences ∇

$$C_\Omega(t) = t\left[-a\left(\nabla^3\delta_{2p} + \Delta^3\delta_0\right) - b\left(\nabla^4\delta_{2p} + \Delta^4\delta_0\right) - c\left(\nabla^5\delta_{2p} + \Delta^5\delta_0\right) - \cdots\right] \qquad \cdots(15)$$

Therefore ,

$$C_\Omega(t) = -at^4\left(\delta_{2p}^{(3)} - \delta_0^{(3)}\right) - bt^6\left(\delta_{2p}^{(4)} - \delta_0^{(4)}\right) - ct^8\left(\delta_{2p}^{(5)} - \delta_0^{(5)}\right) - \cdots \qquad \cdots(16)$$

Where a, b and c are constants.

So the correction limits of NK1 rule is

$$C_\Omega(t) = J - \Omega(t) = \omega_{\Omega1}t^4 + \omega_{\Omega2}t^6 + \omega_{\Omega3}t^8 + \cdots \qquad \cdots(17)$$

Where $\omega_{\Omega1}, \omega_{\Omega2}, \omega_{\Omega3}, \cdots$ are constant do not depend on value of t but depend on the value of the derivatives at ends of integral.

The error formula is dependent if the integrator is continuous in the integration interval.

In order to know the limits of the correction when the integral is improper in one or both ends of the integral supposed that the integral $J=\int_{r_0}^{r_m}\delta(r)dr$ is improper in the lower limit r_0.
It is understood that the above integration can be written in the form:

$$J=\int_{r_0}^{r_m}\delta(r)dr=\int_{r_0}^{r_2}\delta(r)dr+\int_{r_2}^{r_m}\delta(r)dr$$

Since the function $\delta(r)$ defined at r_2, then by using Tylor series at r_2

$$\delta(r)=\left[1+(r-r_2)D+\frac{(r-r_2)^2}{2!}D^2+\frac{(r-r_2)^3}{3!}D^3+\frac{(r-r_2)^4}{4!}D^4+\cdots\right]\delta(r_2)\qquad\cdots(18)$$

Then :

$$\int_{r_0}^{r_2}\delta(r)dr=\left[(r_2-r_0)-\frac{(r_0-r_2)^2}{2}D-\frac{(r_0-r_2)^3}{6}D^2-\frac{(r_0-r_2)^4}{24}D^3-\cdots\right]\delta(r_2)\quad\cdots(19)$$

But $r_2-r_0=2t$, then:

$$\int_{r_0}^{r_2}\delta(r)dr=\left[2t-2t^2D-\frac{4t^3}{3}D^2+\frac{2t^4}{3}D^3-\cdots\right]\delta(r_2)\qquad\cdots(20)$$

By satisfy $\left(r_0+\frac{t}{2}\right)$ instead of r once, satisfy r_1 again and $\left(r_0+\frac{3t}{2}\right)$ third time in formula (18) and collect the result with equation (20) after multiplying by $\frac{2t}{3}$ get:

$$\int_{r_0}^{r_2}\delta(r)dr=\frac{t}{3}\left[4\delta\left(r_0+\frac{t}{2}\right)+4\delta\left(r_0+\frac{3t}{2}\right)-2\delta(r_1)\right]+[at^4D^3+bt^5D^4+ct^6D^5+\cdots]\delta(r_2)$$

Since $E\delta(r)=\delta(r+t)\Rightarrow\delta(r_2)=E\delta(r_1)$, then:

$$\int_{r_0}^{r_2}\delta(r)dr=\frac{t}{3}\left[4\delta\left(r_0+\frac{t}{2}\right)+4\delta\left(r_0+\frac{3t}{2}\right)-2\delta(r_1)\right]+[at^4D^3+bt^5D^4+ct^6D^5+\cdots]E\delta(r_1)$$

$$\int_{r_0}^{r_2}\delta(r)dr=\frac{t}{3}\left[4\delta\left(r_0+\frac{t}{2}\right)+4\delta\left(r_0+\frac{3t}{2}\right)-2\delta(r_1)\right]+[\bar{a}t^4D^3+\bar{b}t^5D^4+\bar{c}t^6D^5+\cdots]\delta(r_1)\cdots(21)$$

The integral $\int_{r_2}^{r_m}\delta(r)dr$ can be calculated by using NK1 rule.
Therefore

$$\int_{r_0}^{r_m}\delta(r)dr=\frac{t}{3}\left[4\sum_{i=1}^{m}\delta\left(r_0+\frac{2i-1}{2}t\right)-2\sum_{i=1}^{\frac{m}{2}}\delta(r_{2i-1})\right]+\omega_{\Omega1}t^4+\omega_{\Omega2}t^6+\omega_{\Omega3}t^8+\cdots$$
$$+[\bar{a}t^4D^3+\bar{b}t^5D^4+\bar{c}t^6D^5+\cdots]\delta(r_1)\quad\cdots(22)$$

This meaning that the limits of the correction when the integral is improper in the lower limit are

$$C_\Omega(t) = \omega_{\Omega 1}t^4 + \omega_{\Omega 2}t^6 + \omega_{\Omega 3}t^8 + \cdots + \left[\bar{a}t^4D^3 + \bar{b}t^5D^4 + \bar{c}t^6D^5 + \cdots\right]\delta(r_1) \qquad \cdots (23)$$

If the integral $J = \int_{r_0}^{r_m} \delta(r)dr$ is improper in the upper limit r_m can be written as follow:

$$J = \int_{r_0}^{r_m} \delta(r)dr = \int_{r_0}^{r_{m-2}} \delta(r)dr + \int_{r_{m-2}}^{r_m} \delta(r)dr$$

The limits of the correction can be proved by the same way that obtained from formula (23).

When the integral $J = \int_{r_0}^{r_m} \delta(r)dr$ is improper in the lower and upper limits write it as follow

$$J = \int_{r_0}^{r_m} \delta(r)dr = \int_{r_0}^{r_2} \delta(r)dr + \int_{r_0}^{r_{m-2}} \delta(r)dr + \int_{r_{m-2}}^{r_m} \delta(r)dr$$

The limits of the correction become

$$C_\Omega(t) = \left[\bar{a}t^4D^3 + \bar{b}t^5D^4 + \bar{c}t^6D^5 + \cdots\right]\delta(r_1) + \left[\bar{\bar{a}}t^4D^3 + \bar{\bar{b}}t^5D^4 + \bar{\bar{c}}t^6D^5 + \cdots\right]\delta(r_{m-1}) \cdots (24)$$

Where $\bar{a}, \bar{b}, \bar{c}, \cdots$ and $\bar{\bar{a}}, \bar{\bar{b}}, \bar{\bar{c}}, \cdots$ are constant depend on a value of the derivatives at ends of interval integral.

Examples

No	Integral	Analytical value	Type of the function of integration
1	$\int_2^3 [x^2e^x + \cos(\pi x)]\, dx$	7.59035839361062	Continuous at [2,3]
2	$\int_0^1 \ln(x - x^2)^{-1} dx$	2.0000000000000	Not defined at {0,1}
3	$\int_0^1 \frac{1-\sqrt{x}}{1+\sqrt{x}} dx$	0.22741127776022	The derivative is not defined at $x = 1$
4	$\int_{-1}^1 \sin^{-1}(x)dx$	1.14159265358979	The derivative is not defined at{−1,1}
5	$\int_1^2 \frac{\sin^2(x)\ln(x)}{e^2} dx$	Unknown	Continuous at [1,2]

Table (2) - Examples

RESULTS

Example 1: The function of integration is continuous for each point at [2, 3]. Therefore, the formula of correction terms for this integral is similar to the formula (17). Applying NK1rule, obtained the results listed in table (3).

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At m=32, the approximated value is correct for six decimal places by using NK1rule while it is equal to analytical value with applying Romberg acceleration when the exponential of t is 10.

m	Value by NK1	Value by R-NK1	Correction terms
2	7.5816858902248		
4	7.5898277131647	7.59037050136074	4
8	7.5903253895244	7.59035837852923	6
16	7.5903563333561	7.59035839361228	8
32	7.5903582648835	7.59035839361062	10

Table (3) – Results of $\int_{2}^{3} [x^2 e^x + \cos(\pi x)]\, dx$

Example 2: This is an improper integral in the lower and upper limits of integration, so the correction terms are $C_\Omega(t) = at + bt^3 + ct^4 + dt^5 + et^6 + \cdots$

By NK1 method, we obtained the results listed in table (4) and found the difference between the approximated value and analytical value is 0.0018050707782 at $m = 256$, but if we using Romberg acceleration with our rule the difference becomes zero when the exponential of t is 8.

m	Value by NK1	Value by R-NK1	Correction terms
2	1.7698704577223		
4	1.8845450665173	1.9992196753124	1
8	1.9422423702605	2.0000425309596	3
16	1.9711191623649	2.0000014254408	4
32	1.9855594522535	1.9999999909918	5
64	1.9927797180277	1.9999999996654	6
128	1.9963898585070	1.9999999999998	7
256	1.9981949292218	2.0000000000000	8

Table (4) – Results of $\int_{0}^{1} \ln(x - x^2)^{-1} dx$

Example 3: This is an improper integral in the upper limit of integration, so the correction terms are

$C_{\Omega}(t) = at^{3/2} + bt^{5/2} + ct^{7/2} + \cdots + At^{4} + Bt^{6} + \cdots$

Applying NK1 mothed, the results in table (5) and found the difference between the approximated value and analytical value is 0.00000410189504 at $m = 512$, but if using Romberg acceleration the two values are equal when the exponential of t is $13/2$.

m	Value by NK1	Value by R-NK1	Correction terms
2	0.21289577695395		
4	0.22192299233226	0.22686014036302	3/2
8	0.22539737773437	0.22740982940451	5/2
16	0.22668463169475	0.22740982940451	7/2
32	0.22715156752034	0.22741121499891	4
64	0.22731893557874	0.22741127670003	9/2
128	0.22737853508099	0.22741127773780	11/2
256	0.22739968444179	0.22741127759833	6
512	0.22740717586518	0.22741127776022	13/2

Table (5) – Results of $\int_{0}^{1} \frac{1-\sqrt{x}}{1+\sqrt{x}} dx$

Example 4: The integral $\int_{-1}^{1} \sin^{-1}(x)dx$ is an improper integral in the lower and upper limits of integration, but since $\sin^{-1} x$ is an even function then

$$\int_{-1}^{1} \sin^{-1} x\, dx = 2\int_{0}^{1} \sin^{-1} x\, dx$$

which is an improper integral in the upper limit of integration, so the correction terms are

$C_{\Omega}(t) = at^{3/2} + bt^{5/2} + ct^{7/2} + \cdots + At^{4} + Bt^{6} + \cdots$

Applying NK1rule, the results listed in table (3). At $m = 512$, the approximated value is correct for four decimal places by using NK1rule while it is equal to analytical value with applying Romberg acceleration when the exponential of t is $13/2$.

m	Value by NK1	Value by R-NK1	Correction terms
2	1.11859059509921		
4	1.13328052536646	1.14131471500872	3/2
8	1.13863387150170	1.14161475354422	5/2
16	1.14054397097037	1.14159244951409	7/2
32	1.14122149023670	1.14159261789646	4
64	1.14146136070173	1.14159265426580	9/2
128	1.14154622292126	1.14159265359871	11/2
256	1.14157623582388	1.14159265358960	6
512	1.14158684867167	1.14159265358979	13/2

Table (6) – Results of $2\int_0^1 \sin^{-1} x\, dx$

Example 5: The analytical value of this integral is unknown and the function of integration is continuous for each point at [1,2]. Therefore, the formula of correction terms for this integral are similar to the formula (17). Can't use the fundamental calculation theorems to evaluate this integral, so, found it by the approximation method NK1 , the results in tables (7). Note that the values converge vertically towards the value of 0.0719861707404131as well as matching the values of integration in the last two rows when $m = 64$ and $m = 128$. Then, the value of integration is correct for sixteen decimal places.

m	$NK1$	4	6	8	10	12	14
2	0.0711484275185725						
4	0.0719354806911932	0.0719879509027012					
8	0.0719830579820863	0.0719862298014791	0.0719862024824121				
16	0.0719859771741395	0.0719861717869430	0.0719861708660774	0.0719861707420918			
32	0.0719861586583180	0.0719861707572633	0.0719861707409192	0.0719861707404283	0.0719861707404267		
64	0.0719861699855308	0.0719861707406783	0.0719861707404151	0.0719861707404131	0.0719861707404131	0.0719861707404131	
128	0.0719861706932368	0.0719861707404172	0.0719861707404131	0.0719861707404131	0.0719861707404131	0.0719861707404131	0.0719861707404131

Table (7) – Results of $\int_1^2 \frac{\sin^2 x \ln(x)}{e^2} dx$

DISCUSSION AND CONCLUSION

This study concluded from the results of the examples whatever the behavior of the function that the calculation of the approximate values of the single integrations by using NK1 mothed are near to the analytical values without using the correction terms, where obtained a precision ranging from 4-10 in a decimal order if the function continuous from $m = 32$ to $m = 128$, also get a precision ranging from 3-8 in a decimal order with an improper integral at the derivative of function or whose integrands are improper at one or both boundary of integration. While all the results are equal to analytical values with applying Romberg acceleration by using the correction terms.

It should be noted that NK1 rule exceeded the accuracy and the number of partial periods used significantly on the best rules of Newton - Cots as shown in table (1) the number of correct decimal places is clearly higher and at partial intervals much lower than the Newton-Cotes rules.

The tables showed that non-neglect of impairment is of great importance in accelerating the approach of values to the exact values of integrations. Thus, NK1rule can be used to calculate single integrations, regardless of the complementary behavior of continuity or impairment.

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